Mark Scheme (Results)

Summer 2018

Pearson Edexcel GCE Further Mathematics
Statistics S3 Paper 6691_01

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\surd$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- * The answer is printed on the paper
- ⌐ The second mark is dependent on gaining the first mark ⌙

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer

Question Number	Scheme	Marks
1. (a)	$r = \frac{S_{ca}}{\sqrt{S_{cc}S_{aa}}} = \frac{47.7625}{\sqrt{34787.5 \times 0.217287}} = 0.549361...$	B1 (1)
(b)	$H_0 : \rho = 0, H_1 : \rho > 0$	B1
	$(0.549 <) 0.6215$ (Not significant Insufficient evidence to reject H_0)	B1
	Insufficient evidence of a positive correlation between the **concentration of a** radioactive **element** and the **amount of dissolved solids** in groundwater.	B1ft (3)
(c)	Sample | *A* | *B* | *C* | *D* | *E* | *F* | *G* | *H* | *c* | 2 | 6 | 4 | 3 | 7 | 1 | 8 | 5 | *a* | 4 | 5 | 1 | 3 | 8 | 2 | 6 | 7 | *d* | -2 | 1 | 3 | 0 | -1 | -1 | 2 | -2 | d^2 | 4 | 1 | 9 | 0 | 1 | 1 | 4 | 4 | 24	M1 M1A1
	Note Reverse ranks $\sum d^2 = 144$	
	$r_s = 1 - \frac{6\sum d^2}{n(n^2-1)} = 1 - \frac{6 \times 24}{8(64-1)} = 0.71428...$	M1A1 (5)
(d)	$H_0 : \rho_s = 0, H_1 : \rho_s > 0$	B1
	$(0.714 >) 0.6429$ (Significant. Reject H_0)	B1
	Evidence of a positive correlation between the **concentration of a** radioactive **element** and the **amount of dissolved solids** in groundwater.	B1ft (3)
(e)	Results of tests suggest (monotonic) **non-linear** relationship or assumptions for PMCC breached i.e. **not** (joint) **normal**.	B1 (1)
		Total 13

Notes

(a) 1st B1 awrt 0.549

(b) 1st B1 Both correct. Require population parameter ρ and one tailed test.
2nd B1 cv 0.6215
3rd B1 Context required. Must mention **concentration of a** radioactive **element** and **amount of dissolved solids**

(c) 1st M1 for an attempt to rank the concentration of a radioactive element **and** the amount of dissolved solids with at least 4 correct for each variable. Allow reverse ranks.
2nd M1 for attempt at d^2 row
1st A1 all correct
3rd M1 for use of the correct formula and an attempt to rank, follow through their $\sum d^2$ if clearly stated
If answer is not correct, a correct expression is required.
A1 awrt 0.714

(d) 1st B1 for both hypotheses in terms of ρ, one tail H_1. Allow use of ρ_s.
Alternative hypothesis compatible with their ranking.
2nd B1 for cv of 0.6429
3rd B1ft for a correct contextualised comment. Must mention **concentration of a** radioactive **element** and the **amount of dissolved solids.** Follow through their r_s and their cv (provided it is |cv| <1)
Don't insist on the word "positive" for a one-tailed test.

(e) B1 for '**non-linear' oe, or 'not normal'**

Question Number	Scheme	Marks
2. (a)	**Record / List** all ticket numbers of **standard** and **premium** tickets	B1
	Use **random numbers** to select a sample of standard and a sample of premium ticket holders i.e. **within strata.**	B1
	Sample sizes in proportion to the no of standard and no of premium ticket holders at the concert.	B1
		(3)
(b)	$H_0 : \mu_p - \mu_s = 6$ oe [p = premium s = standard]	B1
	$H_1 : \mu_p - \mu_s > 6$ oe	B1
	Standard error = $\sqrt{\frac{10^2}{60}+\frac{8^2}{55}} = \left[\sqrt{2.83030...}\right]$ = [1.682…]	M1
	$z = \frac{\pm(23-15-6)}{"\sqrt{\frac{10^2}{60}+\frac{8^2}{55}}"}$	dM1
	= ±1.1888… awrt ±1.19	A1
	cv 5% one tailed = 1.6449	B1
	Not significant, insufficient evidence to reject H_0	dM1
	Insufficient evidence to support the **manager's claim**	A1cso
	or the mean value of merchandise sold to premium ticket holders is NOT more than £6 greater than the mean value of merchandise sold to standard ticket holders.	
		(8)
(c)	**Sample size is large** so **Central Limit Theorem (CLT) applies** so	B1
	do not need to assume merchandise sold has a normal distribution.	dB1
		(2)
		Total 13

Notes

(a)
1st B1 Sampling frame in context. Accept list of all standard and premium ticket holders at the concert.
2nd B1 Use of **random selection** eg simple random sampling **within strata**
3rd B1 Accept description of n_s, n_p.

(b)
1st & 2nd B1 for hypotheses. Accept μ_1, μ_2 or μ_A, μ_B etc if it is clear which is which.
1st M1 for an attempt at se with 3 out of 4 values correct.
Condone switching 10 and 8: $\sqrt{\frac{10^2 \text{ or } 8^2}{60}+\frac{8^2 \text{ or } 10^2}{55}}$
2nd dM1 dependent on 1st M1 for a correct numerator (must have - 6) and ft their se.
1st A1 for awrt 1.19
3rd B1 for ± 1.6449 seen or probability of awrt 0.117, Sign must match their test statistic.
3rd dM1 dep. on 1st M1 for a correct statement based on their normal cv and their test statistic. Ignore their hypotheses. Allow accept H_0 but reject H_1 is M0. Can be implied by correct conclusion.
2nd A1cso for correct comment in context dependent upon all other marks being awarded.
Must mention merchandise, standard and premium ticket holders and 6 or manager and belief or claim
NB Use of cv for difference in means D will have $D = 6 + 1.6449 \times$ s.e. = awrt 8.33 and requires sight of d =8 with a comment for the 3rd M1

(c)
1st B1 for mentioning large samples **and** CLT
2nd dB1 dependent on 1st B1 for stating **no need to** assume normality. Require merchandise sold not mean merchandise sold.

Question Number	Scheme		Marks
3. (a)	$\overline{x} = \hat{\mu} = 1.55$	cao 1.55	B1
	$s^2 = \dfrac{"\sum x^2"-4\times"1.55"^2}{3} = \dfrac{17}{300}$	awrt 0.057	M1A1ftA1
	$\sum x^2 = 9.78$, $"\sum x^2" > 9.61$, $"\sum x^2" \neq (\sum x)^2 = 38.44$		
	Or $s^2 = \dfrac{0.25^2+0.15^2+0.15^2+0.25^2}{3} = \dfrac{17}{300}$		(4)
(b)	$\mathrm{P}\left(\lvert\mu-\hat{\mu}\rvert < 0.1\right) = 0.99$		
	$\dfrac{0.1}{\frac{0.5}{\sqrt{n}}} = 2.5758$	awrt 2.576	M1B1A1ft
	$n = \left(\dfrac{2.5758\times 0.5}{0.1}\right)^2 \ (= 12.879^2 = 165.8...)$		dM1A1ft
	Sample size $(n \geq) 166$		A1 cso
			(6)
			Total 10

Notes

(a)

1st B1 1.55 correct answer only

1st M1 for a correct expression ft their $\overline{x}$

1st A1ft for a fully correct expression ft their $\overline{x}$ only

2nd A1 accept awrt 0.057

(b)

1st M1 $\dfrac{0.1}{\frac{\text{their } s}{\sqrt{n}}} = z$ value. Accept with an inequality in any direction.

1st B1 2.5758

1st A1ft for any equivalent form. Allow ft of $z = 2.326$ or awrt 3.090. Must use 0.5

2nd dM1 for attempt to solve for n dependent on 1st M leading to $n=$

2nd A1 for $\left(\dfrac{2.5758\times 0.5}{0.1}\right)^2$ Allow ft for 135.2… or 238.7…

3rd A1 for 166 cao

Question Number	Scheme	Marks
4. (a)	$2 \times 2.5758 \times \frac{\sigma}{\sqrt{120}} = 0.47027...\sigma$	M1B1A1
		(3)
(b)	$H_0 : \mu = 6 \quad H_1 : \mu \neq 6$	B1
	(Significance level =)10%	B1
	(6 is in the interval so not significant, do not reject H_0) $\mu = 6$	B1
		(3)
(c)	$1.6449 \times \frac{\sigma}{\sqrt{100}} = (6.25 - 5.14) / 2 (= 0.555)$	M1B1
	$\sigma = 3.374...$	A1
		(3)
		Total 9

Notes

(a)

1st M1 Use of $2\,z\frac{\sigma}{\sqrt{n}}$ with $z > 2$

1st B1 2.58 or better

1st A1 awrt 0.47σ

(b)

1st B1 Both hypotheses in terms of μ.

2nd B1 10%

3rd B1 Correct comment leading to accepting H_0

(c)

1st M1 for $z\frac{\sigma}{\sqrt{100}} = 0.555$ oe, using $n = 100$ and where $|z| > 1.5$

1st B1 for 1.6449 or better in an attempt (could be $1.6449\sigma = c$ or even $1.6449\ \sigma^2 = c$)

1st A1 awrt 3.37. Allow awrt 3.38 from use of $z = 1.64$

Question Number		Scheme	Marks
5	(a)	(Let $W =) L - 3C$	B1
		$\mathrm{E}(W) = 2800 - 3\times 1000 = -200$	B1
		$\mathrm{Var}(W) = 650^2 + 3^2 \times 250^2 = 985\,000$	M1A1
		$\mathrm{P}(W > 0) = \mathrm{P}(Z > \frac{200}{\sqrt{985000}}) = \mathrm{P}(Z > 0.20157...)$, = 0.42015 (calc) or 0.4207 (tables)	dM1 A1
			(6)
	(b)	$(F = C_1 + C_2 + ... + C_8 + L_1 + L_2 + L_3)$	
		$\mathrm{E}(F) = 16400$	B1
		$\mathrm{Var}(F) = 8\times 250^2 + 3\times 650^2 = 1767500$	M1A1
		$\mathrm{P}(F > 20\,000) = \mathrm{P}(Z > \frac{20000-16400}{\sqrt{1767500}}) = \mathrm{P}(Z > 2.7078...)$, = 0.003386…(calc) or 0.0035 (tables) or 0.0034 (interpolation)	dM1,A1
			(5)
	(c)	Assume **selection** of cars and lorries is **random**. **Weights** of cars and lorries are **independent.**	B1
			(1)
			Total 12

Notes	
(a)	1st B1 for forming a suitable variable. May be implied by correct variance. 2nd B1 for –200 cao or 200 if their $W = 3C - L$ 1st M1 for attempting Var(W) = Var(L) + $3^2 \times$ Var(C). Condone swapping L and C. 1st A1 for 985 000 cao 2nd M1 dependent upon first M1 for standardising with their –200 and their 985000 2nd A1 awrt 0.420-0.421
(b)	1st B1 for 16400 cao 1st M1 for attempting Var(F) = $8\times$ Var(C) + $3\times$ Var(L) 1st A1 for 1 767 500 cao 2nd M1 dependent upon first M1 for standardising with their 16400 and their 1767500 2nd A1 awrt 0.003-0.004
(c)	Either random selection or independent weights

Question Number	Scheme	Marks
6(a)	H_0 : B(4, 0.5) is a suitable model H_1: B(4, 0.5) is not a suitable model	B1

Even number count	O_i	E_i	$\frac{(O_i-E_i)^2}{E_i}$	$\frac{O_i^2}{E_i}$
0	12	9.375	0.735	15.36
1	45	37.5	1.5	54
2	36	56.25	7.29	23.04
3	39	37.5	0.06	40.56
4	18	9.375	7.935	34.56

	$E_i = 150 \times \mathrm{P}(X = i)$	M1A1
	$\chi^2 = \sum \frac{(O_i-E_i)^2}{E_i}$ or $\chi^2 = \sum \frac{O_i^2}{E_i} - N$	M1A1
	$\chi^2 = 17.52$ or $\chi = 167.52 - 150 = 17.52$ awrt 17.5	A1
	$\nu = 4, \quad \chi_4^2(1\%) = 13.277$	B1, B1ft
	(Reject H_0 ,) B(4,0.5) is not a suitable model or David's claim incorrect.	A1
		(9)
(b)	$\hat{p} = \frac{0\times12 + 1\times45 + 2\times36 + 3\times39 + 4\times18}{4\times150} = 0.51$	M1 A1
		(2)
(c)	$d = 150 \times 6 \times 0.51^2 \times 0.49^2 = 56.205009$ awrt 56.2	M1, A1
	$e = 150 - (8.65 + 36.00 + 39.00 + "d") = 10.144991$ awrt 10.1 or 10.2	B1ft
	or $e = 150 \times 0.51^4 = 10.1478015$	(3)
(d)	H_0 : B(4, p) is a suitable model H_1: B(4, p) is not a suitable model	B1
		(1)
(e)	$\nu = 3, \quad \chi_3^2(1\%) = 11.345$	B1B1ft
	(16.9>11.345) Reject H_0	
	Binomial is not a suitable model or John's claim incorrect or equivalent contextualised statement that rejects the Binomial model.	B1
		(3)
		Total 18

Notes

(a) 1st B1 Accept 'Binomial with $p = 0.5$' replacing 'B(4, 0.5)'

1st M1 for attempt at $E_i = 150 \times \mathrm{P}(X = i)$ with at least 2 values correct.

1st A1 at least 4 E_i correct to 3sf cao. Condone truncation.

2nd M1 for at least 2 correct calculations from 4th or 5th column.
2nd A1 at least 4 correct to 3sf from 4th or 5th column. Condone truncation.
3rd A1 for a test statistic of awrt 17.5 Answer only implies 2ndM1 2ndA1 3rdA1
4th A1 for correct conclusion rejecting binomial model. Condone missing parameters here.
Award provided their test statistic >11.345

(b) 1st M1 for attempting $\hat{p} = \frac{\sum fx}{600}$ with at least 2 values on the numerator correct

1st A1 for 0.51 cao

(c) 1st M1 $d = 150 \times 6 \times (\text{their } \hat{p})^2 \times (1 - \text{their } \hat{p})^2$

1st A1 awrt 56.2

1st B1ft awrt 10.1 or follow from "d"

(d) 1st B1 accept H_0 : Binomial is a suitable model H_1: Binomial is not a suitable model

(e) 1st B1 $\nu = 3$, 2nd B1 11.345, follow through their $\nu \neq$ their value in part (a)

3rd B1 Correct statement rejecting H_0

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